Bound monopoles in the presence of a dilaton

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We study axially symmetric monopoles of both the SU(2) Yang-Mills-Higgs-Dilaton (YMHD) as well as of the SU(2) Einstein-Yang-Mills-Higgs-Dilaton (EYMHD) system. We find that equally to gravity, the presence of the dilaton field can render an attractive phase. We also study the influence of a massive dilaton on the attractive phase in the YMHD system.

I. INTRODUCTION

The Georgi-Glashow model with SU(2) gauge group constitutes the simplest non-abelian gauge field theory in which topological solitons exist : magnetic monopoles [1,2]. It consists of an SU(2) Yang-Mills theory coupled to a Higgs triplet in the adjoint representation of the group and is spontaneously broken by a Higgs potential (we refer to it as to the YMH model in the following). The solutions are characterized by their winding number n, which arises due to topological arguments and is proportional to the magnetic charge of the configuration. The solution with unit topological charge $n = 1$ can be constructed with a spherically symmetric ansatz of the fields. Since this was found to be the unique spherically symmetric solutions [3], the field configurations corresponding to higher values of the topological charge $n > 1$ (the multimonopoles) involve at most axial symmetry and lead to systems of partial differential equations [4,5]. One feature of multimonopoles is their instability : for generic values of the coupling constants of the theory the long ranged repulsion due to the gauge fields cannot be overcome by the short ranged attraction due to the Higgs field. Only in the so-called BPS (Bogomol'nyi-Prasad-Sommerfield) limit [3,6] in which the Higgs field is massless and therefore long ranged, the two interactions exactly compensate [7,8]. The spatial components of the stress-energy tensor were shown to vanish [9] and thus systems of non-interacting monopoles exist.

A few years ago, the YMH model was coupled to Einstein gravity [10] (resulting in a theory labelled EYMH) and the spherically symmetric gravitating monopoles with unit topological charge were constructed. Also studied were the corresponding non-abelian black holes solutions, which violate the "no-hair" conjecture. Quite recently [11], it was demonstrated that bound states of gravitating multimonopoles exist in the EYMH model. Indeed, solving the equations for numerous values of the coupling constants, it was shown that two phases exist. For small values of the Higgs coupling constant, there exists a phase for which the binding energy of the 2-monopole and the 3-monopole is negative, leading to classical solutions bounded by gravity.

On the other hand, it was pointed out [12], that the coupling of the YMH system to a dilaton field (labelled YMHD) renders regular classical solutions that share many properties with that of the EYMH model.

It is therefore natural to check if the coupling to a dilaton field can also lead to systems of bound monopoles. The aim of this paper is to study this question by analyzing the equations of the full EYMHD model incorporating both gravitation and a dilaton field. Our numerical integration of the equations strongly indicate that the analogy between the EYMH and the YMHD models persist also for the multimonopole solutions.

In Sect.II we specify the model and its different components, the axially symmetric ansatz and the relevant rescaling. The boundary conditions are presented in Sect. III. The numerical solutions and their relevant features are discussed in Sect. IV. In particular, we study the effect of the dilaton on both the solutions in flat and curved space and also briefly discuss the implications of a massive dilaton.

II. SU(2) (EINSTEIN-)YANG-MILLS-HIGGS-DILATON THEORY

The action of the Yang-Mills-Higgs-Dilaton (YMHD) theory reads:

$$S = S_M = \int L_M \sqrt{-g} d^4x \ , \tag{1}$$

while for the Einstein-Yang-Mills-Higgs-Dilaton (EYMHD) theory an additional term from the gravity Lagrangian arises:

$$S = S_G + S_M = \int L_G \sqrt{-g} d^4x + \int L_M \sqrt{-g} d^4x \tag{2}$$

The gravity Lagrangian is given by :

$$L_G = \frac{1}{16\pi G} R \tag{3}$$

where G is Newton's constant.
The matter Lagrangian is given in terms of the gauge field A_μ^a, the dilaton field Ψ and the Higgs field Φ^a ($a = 1, 2, 3$):

$$L_M = -\frac{1}{4} e^{2\kappa\Psi} F_{\mu\nu}^a F^{\mu\nu,a} - \frac{1}{2} \partial_\mu \Psi \partial^\mu \Psi - \frac{1}{2} D_\mu \Phi^a D^\mu \Phi^a - e^{-2\kappa\Psi} V(\Phi^a) - \frac{1}{2} m^2 \Psi^2 \tag{4}$$

where m denotes the mass of the dilaton field and

$$V(\Phi^a) = \frac{\lambda}{4} (\Phi^a \Phi^a - v^2)^2 \tag{5}$$

The field strength tensor is given by:

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + e\varepsilon_{abc} A_\mu^b A_\nu^c \tag{6}$$

and the covariant derivative of the in the adjoint representation given Higgs field reads:

$$D_\mu \Phi^a = \partial_\mu \Phi^a + e\varepsilon_{abc} A_\mu^b \Phi^c \tag{7}$$

e denotes the gauge field coupling, κ the dilaton coupling, λ the Higgs field coupling and v the vacuum expectation value of the Higgs field.

III. AXIALLY SYMMETRIC ANSATZ

For the metric, the axially symmetric Ansatz in isotropic coordinates reads:

$$ds^2 = -f dt^2 + \frac{m}{f} \left(dr^2 + r^2 d\theta^2 \right) + \frac{l}{f} r^2 \sin^2\theta d\varphi^2 \ , \tag{8}$$

where f, m and l are functions of r and θ only. In the special case of the YMHD system $m(r,\theta) = l(r,\theta) = f(r,\theta) = 1$.
The ansatz for the purely magnetic gauge field is

$$A_\mu dx^\mu = \frac{1}{2} A_\mu^a \tau^a dx^\mu = \frac{1}{2er} \left[\tau_\phi^n \left(H_1 dr + (1 - H_2) r d\theta \right) - n \left(\tau_r^n H_3 + \tau_\theta^n (1 - H_4) \right) r \sin\theta d\varphi \right] \ , \tag{9}$$

and for the Higgs field the ansatz reads

$$\Phi = \Phi^a \tau^a = (\Phi_1 \tau_r^n + \Phi_2 \tau_\theta^n) \tag{10}$$

where the matter field functions H_1, H_2, H_3, H_4, Φ_1 and Φ_2 depend only on r and θ. The symbols τ_r^n, τ_θ^n and τ_ϕ^n denote the dot products of the cartesian vector of Pauli matrices, $\vec{\tau} = (\tau^1, \tau^2, \tau^3)$, with the spatial unit vectors

$$\begin{aligned}
\vec{e}_r^{\,n} &= (\sin\theta\cos n\varphi, \sin\theta\sin n\varphi, \cos\theta) \ , \\
\vec{e}_\theta^{\,n} &= (\cos\theta\cos n\varphi, \cos\theta\sin n\varphi, -\sin\theta) \ , \\
\vec{e}_\phi^{\,n} &= (-\sin n\varphi, \cos n\varphi, 0) \ ,
\end{aligned} \tag{11}$$

respectively. Here, the topological charge n enters the Ansatz for the fields.
The dilaton field Ψ is a scalar field depending on r, θ:

$$\Psi = \Psi(r,\theta) \tag{12}$$

A. Rescaling

With the introduction of the dimensionless radial coordinate x and rescaling of the Higgs field, the dilaton field and the dilaton mass, respectively:

$$x \equiv rev \ , \ \phi = \frac{\Phi}{v} \ , \ \psi = \frac{\Psi}{v} \ , \ M_{dil} = \frac{m}{ev} \ , \tag{13}$$

the set of partial differential equations depends only on three fundamental coupling constants:

$$\alpha = \sqrt{4\pi G}v, \quad \beta = \frac{\sqrt{\lambda}}{e}, \quad \gamma = v\kappa \tag{14}$$

where $\alpha = 0$ in the YMHD system.

B. Mass of the solution

In the case of a massive dilaton ($M_{dil} \neq 0$), the mass of the solution μ can be obtained from integrating the Lagrangian density (4). For $M_{dil} = 0$, however, simple relations between the mass of the solution and the derivative of the corresponding function at infinity exist. In the YMHD system the mass is given in terms of the derivative of the dilaton field at infinity [12]

$$\mu = \frac{1}{\gamma} \lim_{x\to\infty} x^2 \partial_x \psi \tag{15}$$

while in the EYMHD system it is given in terms of the derivative of the metric function f at infinity

$$\mu = \frac{1}{2\alpha^2} \lim_{x\to\infty} x^2 \partial_x f \tag{16}$$

The mass μ_{ab} of the corresponding abelian solutions is given in the EYMHD system by:

$$\mu_{ab} = (\alpha^2 + \gamma^2)^{-1/2} \tag{17}$$

with $\alpha = 0$ in the limit of the YMHD system.

IV. BOUNDARY CONDITIONS

We look for regular, static, finite energy solutions that are asymptotically flat. The requirement of regularity leads to the following boundary conditions at the origin:

$$\partial_x f(0,\theta) = \partial_x l(0,\theta) = \partial_x m(0,\theta) = 0, \quad \partial_x \psi(0,\theta) = 0 \tag{18}$$

$$H_i(0,\theta) = 0, \ i = 1,3, \quad H_i(0,\theta) = 1, \ i = 2,4, \quad \phi_i(0,\theta) = 0, \ i = 1,2 \tag{19}$$

At infinity, the requirement for finite energy and asymptotically flat solutions leads to the boundary conditions:

$$f(\infty,\theta) = l(\infty,\theta) = m(\infty,\theta) = 1, \quad \psi(\infty,\theta) = 0 \tag{20}$$

$$H_i(\infty,\theta) = 0, \ i = 1,2,3,4, \quad \phi_1(\infty,\theta) = 1, \quad \phi_2(\infty,\theta) = 0 \tag{21}$$

In addition, boundary conditions on the symmetry axes (the ρ- and z-axes) have to be fulfilled. On both axes:

$$H_1 = H_3 = \phi_2 = 0 \tag{22}$$

and

$$\partial_\theta f = \partial_\theta m = \partial_\theta l = \partial_\theta H_2 = \partial_\theta H_4 = \partial_\theta \phi_1 = \partial_\theta \psi = 0 \tag{23}$$

V. NUMERICAL RESULTS

Subject to the above boundary conditions, we have solved the system of partial differential equations numerically.

A. Monopoles in YMHD theory

It was noted recently [11] that in a certain parameter range of the coupling constants an attractive phase exists in the EYMH system. Inspired by the observation that the monopoles in the YMHD system share many features with the monopoles in the EYMH system [12], we first studied the (multi)monopoles of the YMHD system in the limit of vanishing dilaton mass. We find that in the BPS limit ($\beta = 0$) there exists an attractive phase for all values of $\gamma > 0$. This is in close analogy to the EYMH system, where attraction between the BPS monopoles exists for all $\alpha > 0$. Indeed, the plot of the energy per winding number over γ in the YMHD system looks similar than Fig. 3 of [11] when α is interchanged with γ and "Reissner-Nordström (RN)" is interchanged with "Einstein-Maxwell-Dilaton (EMD)". Moreover, we find that when comparing the quantity

$$\Delta E = \frac{E(n=1)}{1} - \frac{E(n=2)}{2} \tag{24}$$

for the monopoles in the EYMH system for a specific value of $\tilde{\alpha}$ with that of the monopoles in the YMHD system for a value of $\gamma = \tilde{\alpha}$, the two values equal each other (at least within our numerical accurancy).

We were also interested in the implications of a massive dilaton. The massive dilaton was previously considered only for the spherically symmetric solutions [13]. We studied the influence of the dilaton mass M_{dil} on the attractive phase. Since now a mass is involved, the dilaton field decays exponentially - contrasted to a power law decay in the massless case - and the relation (15) between the derivative of the dilaton field at infinity and the mass of the solution is no longer valid. Our numerical results are shown in Fig. 1, where we present the difference between the mass (per winding number) of the $n = 1$ solution and the mass per winding number of the $n = 2$ solution $\Delta E = \frac{E(n=1)}{1} - \frac{E(n=2)}{2}$. Clearly, the attraction is lost for $M_{dil} > \hat{M}_{dil}(\gamma)$. We find that $\hat{M}_{dil}(\gamma = 0.5) \approx 0.040$ and $\hat{M}_{dil}(\gamma = 1.0) \approx 0.028$, respectively. Our numerical results suggest further that for $M_{dil} \to \infty$ the value ΔE turns to zero indicating that the monopoles are non-interacting in this limit. This is demonstrated in the following table for $\gamma = 1.0$:

M_{dil}	ΔE
0.	0.00737
0.01	0.00372
0.1	-0.01390
1.	-0.01380
10.	-0.00023

This result can be understood considering that for $M_{dil} \to \infty$ the dilaton function $\psi(x, \theta)$ has to turn to zero on the full interval $x\epsilon$ $[0 : \infty[$ for all θ. Thus for the case studied here ($\beta = 0$), the BPS limit of the YMH system is recovered for $M_{dil} \to \infty$. Our numerical results strongly support this interpretation. We find that with increasing M_{dil} the dilaton field tends more and more to the trivial solution $\psi(x, \theta) = 0$ and that the mass tends to one, which (in our rescaled variables) is just the mass of the BPS solution in the YMH system.

B. Monopoles in EYMHD theory

Here, we only considered the case of $M_{dil} = 0$. We first studied the influence of the dilaton field on the attraction between like monopoles in the limit of vanishing Higgs coupling (BPS limit). In Fig. 2, we show the difference between the mass per winding number of the $n = 1$ and the $n = 2$ solution $\Delta E = \frac{E(n=1)}{1} - \frac{E(n=2)}{2}$ for a fixed α and varying γ. For $\alpha = 0$ the limit $\gamma = 0$ represents the BPS limit of the YMH theory. The monopoles are non-interacting for $\beta = 0$ and therefore the energy per winding number is equal for all (multi)monopole solutions of different topological sectors. Since the YMHD-monopoles in the BPS limit reside in an attractive phase for all $\gamma \neq 0$, ΔE should be positive, which indeed, is demonstrated in Fig. 2. For $\alpha = 0.5$ the attraction between like monopoles in the EYMHD system is bigger than in the pure EYMH system ($\gamma = 0$) for all $\gamma \neq 0$. The curve reaches a maximum of the difference at some value γ and from there, the difference gets smaller. This can be understood from the fact that for rising γ, the solutions tend to the EMD solutions which have mass per winding number equal for all n. The curve for $\alpha = 1.0$ shows the same behaviour apart from the fact, that now for bigger γ the attraction gets smaller than in the $\gamma = 0$ case. This is due to the fact, that in the pure EYMH system, the attraction has nearly reached its maximum at $\alpha = 1.0$ and that now inclusion of the dilaton field very soon makes the solution tend to a EMD solution.

To study the influence of the dilaton on the monopole solutions for $\beta \neq 0$, we followed [11] and determined $\gamma_{eq}(\beta)$. This is - for a fixed β - the value of γ for which the mass of the $n = 1$ solution is equal to the mass per winding number of the $n = 2$ solution. For $\gamma < \gamma_{eq}$, the mass per winding number of the $n = 2$ is bigger than the mass of the $n = 1$ solution which implies that the monopoles are repelling, while for $\gamma > \gamma_{eq}$ it is smaller leading to an attractive phase. Because globally regular solutions exist only for $\gamma \leq \gamma^{n}_{max}(\beta)$ [14], the attractive phase is limited in parameter space by the $\gamma^{n=1}_{max}$ curve. (Since $\gamma^{n=2}_{max}(\beta) > \gamma^{n=1}_{max}(\beta)$ for the values of β for which the attractive phase exists, the masses of the $n = 1$ and $n = 2$ solution can only be compared for $\gamma \leq \gamma^{n=1}_{max}$.) For $\beta = \hat{\beta}$, the two curves meet and no attractive phase is possible for $\beta > \hat{\beta}$. In the EYMH system it was found that $\hat{\beta} \approx 0.21$ [11]. In Fig. 3, the values of γ_{eq} and γ_{max} are shown for three different values of α. $\alpha = 0$ represents the YMHD system and the γ_{max}- and γ_{eq}- curves look similar than the α_{max}- and α_{eq}- curves of [11]. This again underlines the close analogy of the EYMH system and the YMHD system.

Comparing the three curves, we find that the values of both γ_{max} and γ_{eq} drop to smaller values of γ for fixed β and increasing α. This results in the fact that the value of $\hat{\beta}$ seems to be independent on α. For α_1 the attractive phase is thus obtained for smaller values of γ then in the α_2 case, if $\alpha_1 > \alpha_2$. It does not seem to exceed $\beta > 0.21$ for any value of α though.

VI. SUMMARY

We have studied axially symmetric dilatonic monopoles in flat and curved space. In the limit of vanishing gravitational coupling and vanishing dilaton mass, we find that the presence of the dilaton field can render an attractive phase similar to gravity. The close analogy between the EYMH and the YMHD system observed in [12] thus persists for the multimonopoles. When the dilaton field is massive, the attraction between the monopoles in the BPS limit of the YMHD system is lost for $M_{dil} > \hat{M}_{dil}(\gamma)$ and the monopoles are repelling. For $M_{dil} \to \infty$, the dilaton function has to turn to the trivial solution (to fulfill the requirement of finite energy). The dilaton decouples from the field equations and the pure YMH system, in which the BPS monopoles are known to be non-interacting, is left.

When both gravitation and the (massless) dilaton are coupled to the BPS monopoles, the value of $\Delta E = \frac{E(n=1)}{1} - \frac{E(n=2)}{2}$ - indicating the strength of attraction - first increases from its value in the EYMH system with increasing γ. ΔE reaches a maximum at a value of γ depending on α and from there decreases.

In the non-BPS limit, the (massless) dilaton field is able to overcome the long ranged repulsion of the gauge fields in a similar than gravity. We find that the attractive phase is limited in parameter space and that the value of β for which the attractive phase is lost is independent on α.

While the $n = 1$ monopole is stable due to the preservation of the topological charge, the stability of $n > 1$ monopoles is not obvious since it might be possible that they decay into singly charged monopoles thereby preserving the total topological charge. We conjecture that the monpoles are stable as long as they reside in the attractive phase.

We have studied axially symmetric monopoles for $n = 2$ here. However, it was observed that for $n \geq 3$ BPS monopoles with discrete symmetries exist [15]. Since in the BPS limit the energy per winding number is equal for all configurations (independent on the actual structure), it would be interesting to construct these solutions in the (E)YMHD system. Only then it could be decided, which configuration is the one of lowest energy for a given topological sector.

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Figure 1

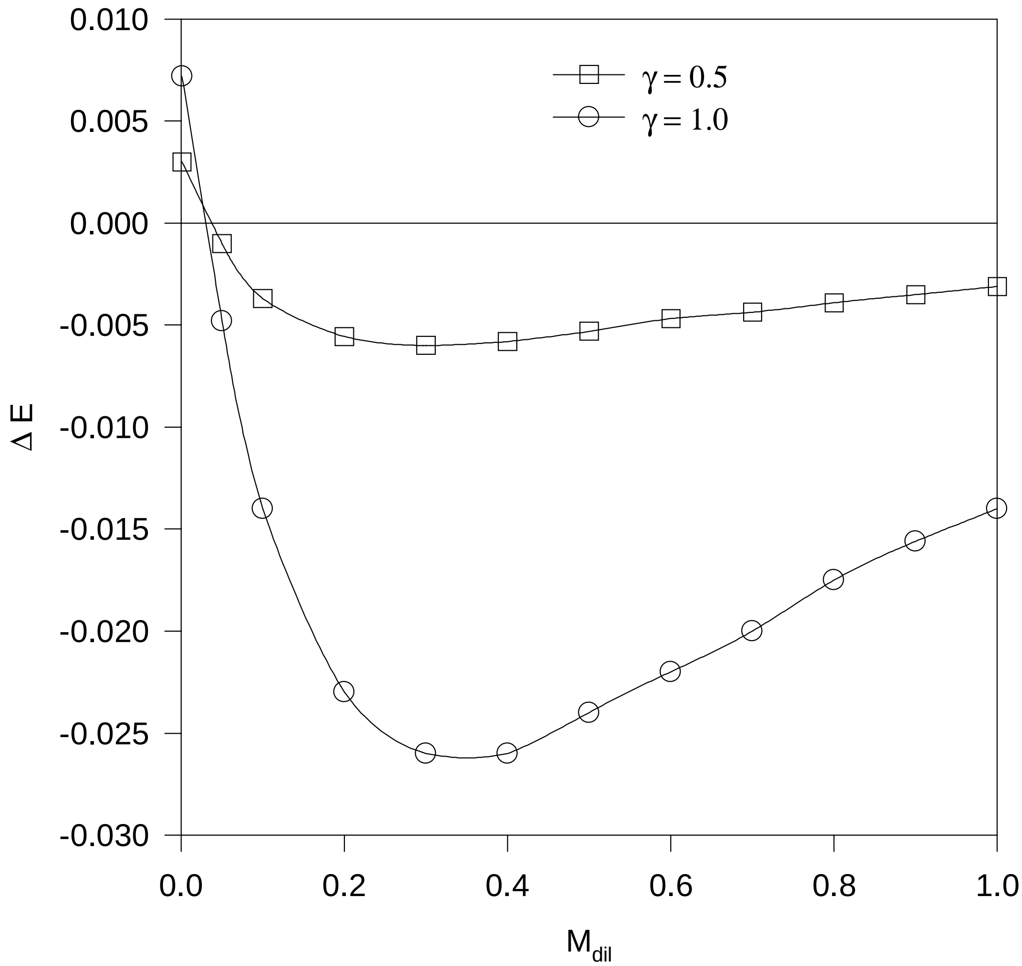

FIG. 1. The quantity $\Delta E = \frac{E(n=1)}{1} - \frac{E(n=2)}{2}$ is shown as function of the dilaton mass M_{dil} for two different values of γ in the YMHD system ($\alpha = 0$).

Figure 2

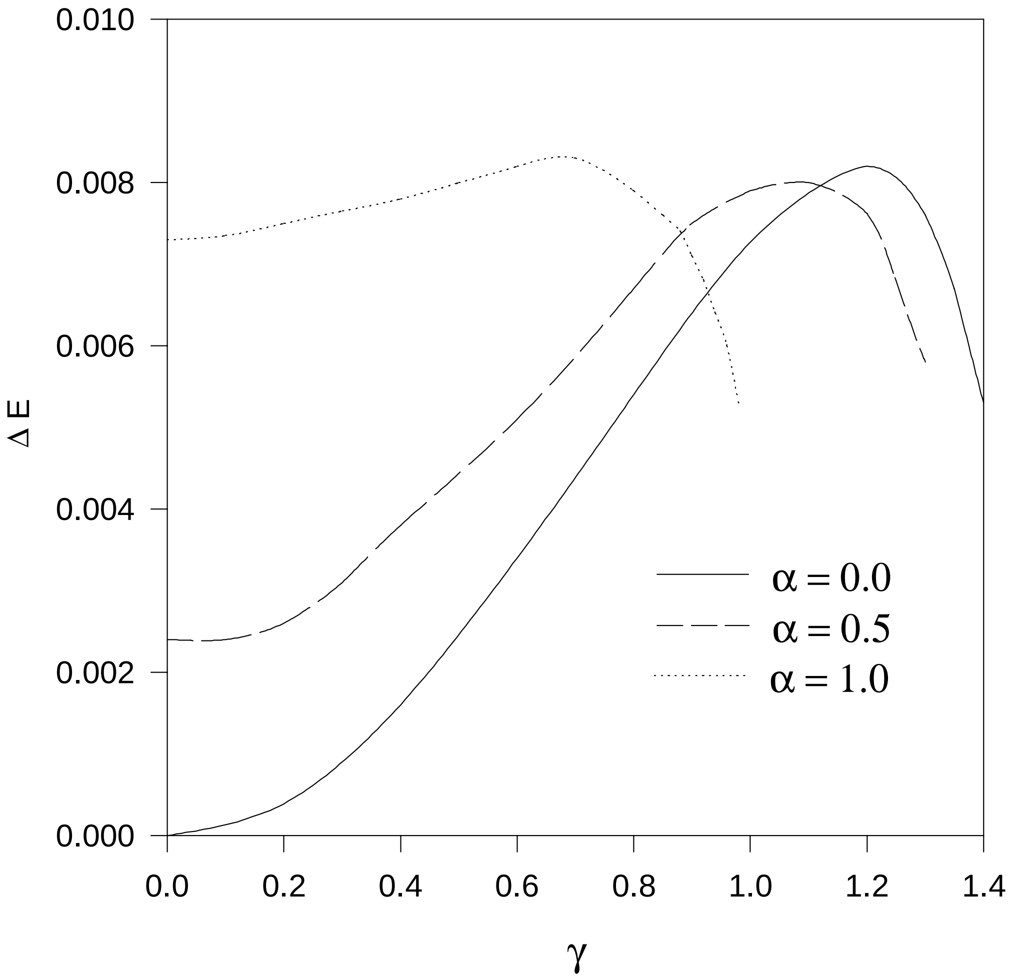

FIG. 2. The quantity $\Delta E = \frac{E(n=1)}{1} - \frac{E(n=2)}{2}$ is shown as function of γ for three different values of α, including $\alpha = 0.0$, which represents the YHMD system.

Figure 3

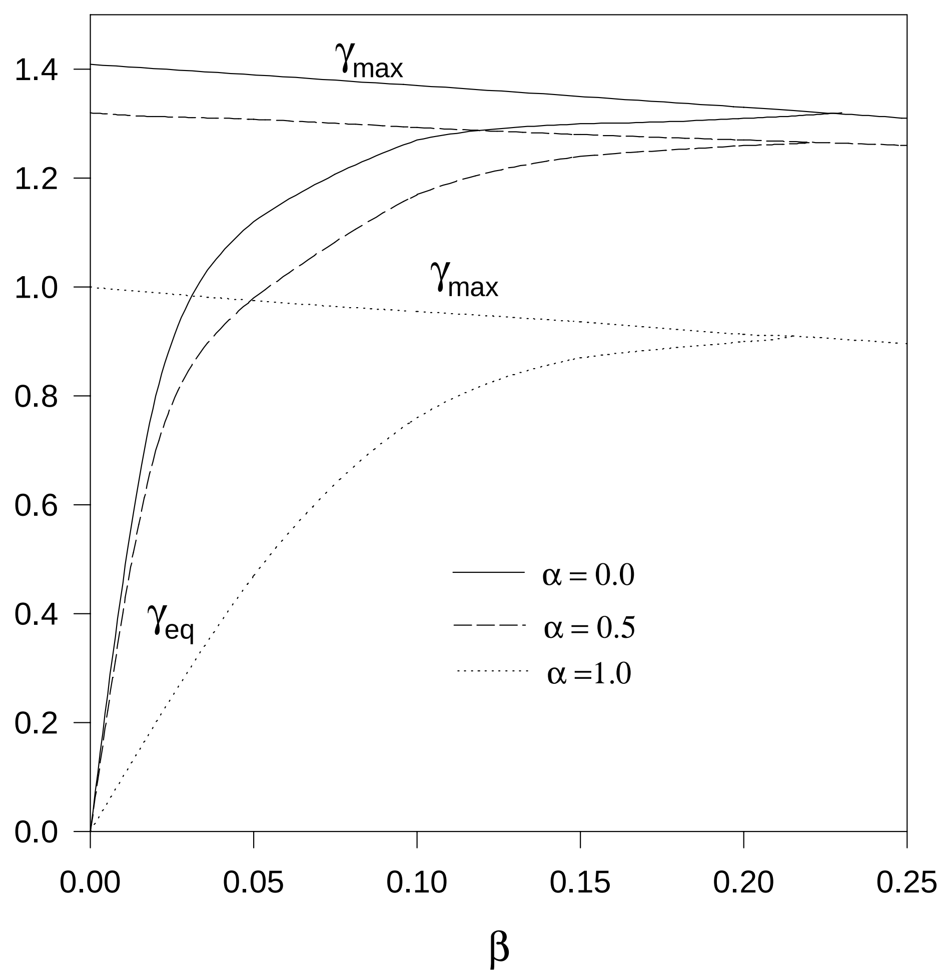

FIG. 3. γ_{eq} is shown as a function of β for three different values of α. Also shown is $\gamma_{max}^{n=1}$. The attractive phase exists for parameters values above the γ_{eq} curve and below the corresponding $\gamma_{max}^{n=1}$ line.